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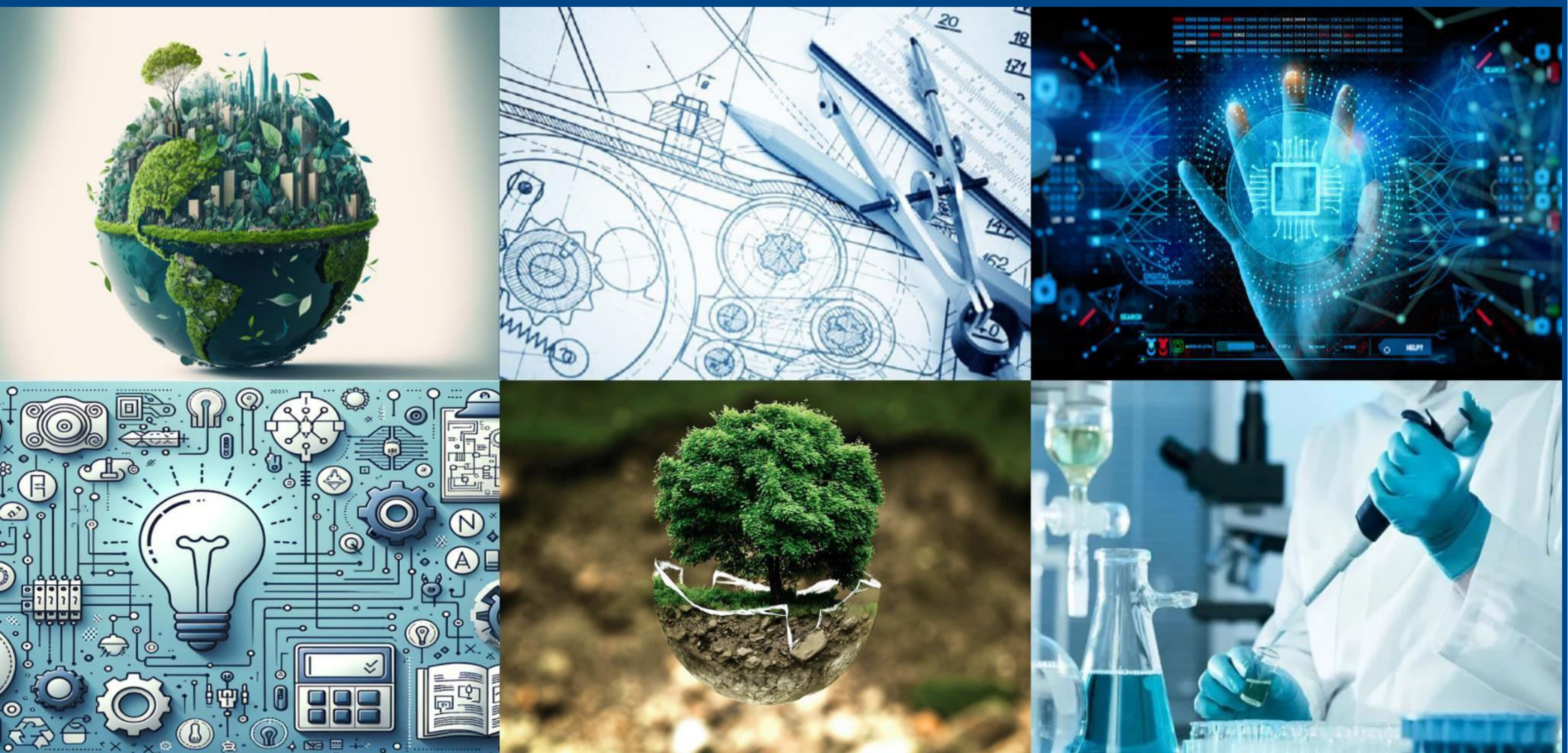

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Interval-Valued Picture Fuzzy BG-Subalgebra in BG-algebra

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ABSTRACT: In this article, we introduce the notion of Interval valued Picture Fuzzy BG-Subalgebra in BG-algebra with example by extending the concept of picture fuzzy BG-Subalgebra. We discuss some of their properties.

KEYWORDS: BG-algebra, Picture Fuzzy set, Picture Fuzzy BG-Subalgebra, Interval Valued Picture Fuzzy BG-Subalgebra**.**

I. INTRODUCTION

In 2014, Cuong [5] introduced picture fuzzy sets, expanding on fuzzy sets and intuitionistic fuzzy sets. This concept is particularly useful in scenarios involving diverse responses, such as yes, no, abstain, and rejection. Picture fuzzy sets evaluate positive, neutral, and negative membership degrees, providing a broader framework. Kim and Kim proposed BG-algebras, a generalization of B-algebras. Ahn and Lee [3] later introduced fuzzy subalgebras in BG-algebras in 2004. P. Naga Sriveni and K. Lalitha Parameswari [7] introduced the concept of Picture Fuzzy BG-Subalgebra in BG-Subalgebra. In this study we extend this concept to interval valued picture fuzzy BG-subalgebra in BG-algebra by utilizing interval valued picture fuzzy set. we provide some of their properties and characteristics

II. LITERATURE REVIEW

Definition 2.1 [1] A non-empty set $\mathcal{X}$ with a constant 0 and a binary operation $\bowtie$ is said to be BG-Algebra if it satisfies the following axioms

1. $x_{01} \bowtie x_{01} = 0$
2. $x_{01} \bowtie 0 = 0$
3. $(x_{01} \bowtie x_{02}) \bowtie (0 \bowtie x_{02}) = x_{01}$ for all $x_{01}, x_{02} \in \mathcal{X}$.

Definition 2.2 [1] A non-empty subset $\mathcal{I}$ of a BG-algebra $\mathcal{X}$ is called a subalgebra of $\mathcal{X}$ if $x_{01} \bowtie x_{02} \in \mathcal{I}$ for all $x_{01}, x_{02} \in \mathcal{I}$.

Definition 2.3 [2] Let $\mathcal{X}$ be the collection of objects. Then a fuzzy set $\mathcal{F}$ in $\mathcal{X}$ is defined as
$\mathcal{F} = \{(x, \alpha(x)) \mid x \in \mathcal{X}\}$, where $\alpha(x)$ is called the membership degree of x in $\mathcal{F}$ and $0 \le \alpha(x) \le 1$.

Definition 2.5 [3] A fuzzy set $\mathcal{F}$ is said to be a fuzzy subalgebra of $\mathcal{X}$ if $\alpha(x) \ge min\{\alpha(x * y), \alpha(y)\}$ for all $x, y \in \mathcal{X}$.

Definition 2.5 [4] An Intuitionistic fuzzy set $\mathfrak{B}$ in a non-empty set $\mathcal{X}$ is an object having the form $\mathfrak{B} = \{(x, \alpha(x), \beta(x)) | x \in \mathcal{X}\}$ where $\alpha(x), \beta(x)$ are degree of belongingness and degree of non-belongingness of $x \in \mathcal{X}$ respectively and $0 \le \alpha(x) + \beta(x) \le 1$ for all $x \in \mathcal{X}$.

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Definition 2.6 [6] Let $\mathcal{X}$ be a non-empty and finite set. An interval valued picture fuzzy set in $\mathcal{X}$ is defined by

$$\bar{\bar{\mathcal{N}}} = \{\langle x_{01}, \alpha^I(x_{01}), \beta^I(x_{01}), \gamma^I(x_{01})\rangle | x_{01} \in \mathcal{X}\}$$

Where $\alpha^I: \mathcal{X} \rightarrow$ I[0,1], $\beta^I: \mathcal{X} \rightarrow$ I[0,1] and $\gamma^I: \mathcal{X} \rightarrow$ I[0,1] positive, neutral, and negative interval valued membership functions respectively, and $[0,0] \ll \alpha^I(x_{01}) + \beta^I(x_{01}) + \gamma^I(x_{01}) \ll [1,1]$. Furthermore, $H(x_{01}) = [1,1] - \alpha^I(x_{01}) - \beta^I(x_{01}) - \gamma^I(x_{01})$ is the refusal membership function.

Definition 2.7. [7] A Picture fuzzy set $\mathcal{N} = (\alpha, \beta, \gamma)$ in $\mathcal{X}$ is called a Picture Fuzzy BG-Subalgebra if it satisfies the following conditions $\alpha(x * y) \geq min\{\alpha(x), \alpha(y)\}$; $\beta(x * y) \geq min\{\beta(x), \beta(y)\}$; $\gamma(x * y) \leq max\{\gamma(x), \gamma(y)\}$ for all $x, y \in \mathcal{X}$.

III. MAIN RESULTS

Definition 3.1. An interval valued picture fuzzy set $\bar{\bar{\mathcal{N}}} = (\alpha^I, \beta^I, \gamma^I)$ in $\mathcal{X}$ is called an interval valued picture fuzzy BG-Subalgebra if it satisfies

(IvPFBG-SA 1) $\alpha^I(x_{01} \bowtie x_{02}) \gg rmin\{\alpha^I(x_{01}), \alpha^I(x_{02})\}$

(IvPFBG-SA 2) $\beta^I(x_{01} \bowtie x_{02}) \gg rmin\{\beta^I(x_{01}), \beta^I(x_{02})\}$

(IvPFBG-SA 3) $\gamma^I(x_{01} \bowtie x_{02}) \ll rmax\{\gamma^I(x_{01}), \gamma^I(x_{02})\}$

for all $x_{01}, x_{02} \in \mathcal{X}$.

Definition 3.2. Consider a set $\mathcal{X} = \{0,1,2,3,4,5\}$ with the binary operation '⋈' which is given in table.1

⋈	0	a	b	c	d	e
0	0	e	d	c	b	a
a	a	0	e	d	c	b
b	b	a	0	e	d	c
c	c	b	a	0	e	d
d	d	c	b	a	0	e
e	e	d	c	b	a	0

Table:1

Then $(\mathcal{X}; \bowtie, 0)$ is a BG-algebra. Let $\bar{\bar{\mathcal{N}}} = (\alpha^I, \beta^I, \gamma^I)$ be a picture fuzzy set in $\mathcal{X}$ defined by table.2

$\mathcal{X}$	0	a	b	c	d	e
$\alpha^I(x_{01})$	[0.4, 0.8]	[0.39,0.7]	[0.4, 0.8]	[0.39,0.7]	[0.4, 0.8]	[0.39,0.7]
$\beta^I(x_{01})$	[0.29, 0.73]	[0.23,0.52]	[0.29, 0.73]	[0.23,0.52]	[0.29, 0.73]	[0.23,0.52]
$\gamma^I(x_{01})$	[0.09, 0.31]	[0.2,0.4]	[0.09, 0.31]	[0.2,0.4]	[0.09, 0.31]	[0.2,0.4]

Table:2

It is routine to verify that $\bar{\bar{\mathcal{N}}} = (\alpha^I, \beta^I, \gamma^I)$ is an interval valued picture fuzzy BG-Subalgebra of $\mathcal{X}$.

Proposition 3.3. If $\bar{\bar{\mathcal{N}}} = (\alpha^I, \beta^I, \gamma^I)$ in $\mathcal{X}$ is an interval valued picture fuzzy BG-Subalgebra in $\mathcal{X}$, then for all $x_{01} \in \mathcal{X}$ $\alpha^I(0) \gg \alpha^I(x_{01}), \beta^I(0) \gg \beta^I(x_{01})$ and $\gamma^I(0) \ll \gamma^I(x_{01})$.

Proof: Let $x_{01} \in \mathcal{X}$. Then

$$\alpha^I(0) = \alpha^I(x_{01} \bowtie x_{01}) \gg rmin\{\alpha^I(x_{01}), \alpha^I(x_{01})\} = \alpha^I(x_{01})$$
$$\Rightarrow \alpha^I(0) \gg \alpha^I(x_{01}),$$
$$\beta^I(0) = \beta^I(x_{01} \bowtie x_{01}) \gg rmin\{\beta^I(x_{01}), \beta^I(x_{01})\} = \beta^I(x_{01})$$
$$\Rightarrow \beta^I(0) \gg \beta^I(x_{01}),$$

$$\gamma^I(0) = \gamma^I(x_{01} \bowtie x_{01}) \ll rmax\{\gamma^I(x_{01}), \gamma^I(x_{01})\} = \gamma^I(x_{01})$$
$$\Rightarrow \gamma^I(0) \ll \gamma^I(x_{01}).$$

Theorem 3.4. Let $\bar{\bar{\mathcal{N}}} = (\alpha^I, \beta^I, \gamma^I)$ be an interval valued picture fuzzy BG-Subalgebra of $\mathcal{X}$. If there exists a sequence $\{x_{01_n}\}$ in $\mathcal{X}$ such that $\lim_{n\to\infty} \alpha^I(x_{01_n}) = [1,1]$, $\lim_{n\to\infty} \beta^I(x_{01_n}) = [1,1]$ and $\lim_{n\to\infty} \gamma^I(x_{01_n}) = [0,0]$, then $\alpha^I(0) = [1,1]$, $\beta^I(0) = [1,1]$ and $\gamma^I(0) = [0,0]$.

Proof: Using the proposition 3.3, we know that $\alpha^I(0) \gg \alpha^I(x_{01_n})$ $\beta^I(0) \gg \beta^I(x_{01_n})$ and $\gamma^I(0) \ll \gamma^I(x_{01_n})$ for every positive integer n. Note that

$[1,1] \gg \alpha^I(0) \gg \lim_{n\to\infty} \alpha^I(x_{01_n}) = [1,1]$

$[1,1] \gg \beta^I(0) \gg \lim_{n\to\infty} \beta^I(x_{01_n}) = [1,1]$

$[0,0] \ll \gamma^I(0) \ll \lim_{n\to\infty} \gamma^I(x_{01_n}) = [0,0]$

Therefore $\alpha^I(0) = [1,1]$, $\beta^I(0) = [1,1]$ and $\gamma^I(0) = [0,0]$.

Proposition 3.5. If an interval valued picture fuzzy set $\bar{\bar{\mathcal{N}}} = (\alpha^I, \beta^I, \gamma^I)$ in $\mathcal{X}$ be an interval valued picture fuzzy BG-Subalgebra, then for all $x_{01} \in \mathcal{X}$ $\alpha^I(0 \bowtie x_{01}) \gg \alpha^I(x_{01}), \beta^I(0 \bowtie x_{01}) \gg \beta^I(x_{01})$ and $\gamma^I(0 \bowtie x_{01}) \ll \gamma^I(x_{01})$.

Proof: For all $x_{01} \in \mathcal{X}$,
we have

$$\begin{aligned}\alpha^I(0 \bowtie x_{01}) &\gg rmin\{\alpha^I(0), \alpha^I(x_{01})\}\\ &= rmin\{\alpha^I(x_{01} \bowtie x_{01}), \alpha^I(x_{01})\}\\ &\gg rmin\{rmin\{\alpha^I(x_{01}), \alpha^I(x_{01})\}, \alpha^I(x_{01})\}\\ &= \alpha^I(x_{01})\end{aligned}$$
$$\Rightarrow \alpha^I(0 \bowtie x_{01}) \gg \alpha^I(x_{01}),$$

$$\begin{aligned}\beta^I(0 \bowtie x_{01}) &\gg rmin\{\beta^I(0), \beta^I(x_{01})\}\\ &= rmin\{\beta^I(x_{01} \bowtie x_{01}), \beta^I(x_{01})\}\\ &\gg rmin\{rmin\{\beta^I(x_{01}), \beta^I(x_{01})\}, \beta^I(x_{01})\}\\ &= \beta^I(x_{01})\end{aligned}$$
$$\Rightarrow \beta^I(0 \bowtie x_{01}) \gg \beta^I(x_{01}),$$

$$\begin{aligned}\gamma^I(0 \bowtie x_{01}) &\ll rmax\{\gamma^I(0), \gamma^I(x_{01})\}\\ &= rmax\{\gamma^I(x_{01} \bowtie x_{01}), \gamma^I(x_{01})\}\\ &\ll rmax\{rmax\{\gamma^I(x_{01}), \gamma^I(x_{01})\}, \gamma^I(x_{01})\}\\ &= \gamma^I(x_{01})\end{aligned}$$
$$\Rightarrow \gamma^I(0 \bowtie x_{01}) \ll \gamma^I(x_{01}).$$

Definition 3.6. Let $\bar{\bar{\mathcal{N}}}_1 = \left(\alpha^I{}_1, \beta^I{}_1, \gamma^I{}_1\right)$ *and* $\bar{\bar{\mathcal{N}}}_2 = \left(\alpha^I{}_2, \beta^I{}_2, \gamma^I{}_2\right)$ be two interval valued picture fuzzy sets, then the intersection is defined as

$$\bar{\bar{\mathcal{N}}}_1 \cap \bar{\bar{\mathcal{N}}}_2 = \left\{\left(x_{01}, rmin\left(\alpha^I{}_1(x_{01}), \alpha^I{}_2(x_{01})\right), rmin\left(\beta^I{}_1(x_{01}), \beta^I{}_2(x_{01})\right), rmax\left(\gamma^I{}_1(x_{01}), \gamma^I{}_2(x_{01})\right)\right) : x_{01} \in \mathcal{X}\right\}$$

Theorem 3.7. Let $\bar{\bar{\mathcal{N}}}_1$ and $\bar{\bar{\mathcal{N}}}_2$ be two interval valued picture fuzzy BG-Subalgebras of $\mathcal{X}$, then $\bar{\bar{\mathcal{N}}}_1 \cap \bar{\bar{\mathcal{N}}}_2$ is an interval valued picture fuzzy BG-Subalgebra of $\mathcal{X}$.

Proof: Let $x_{01}, x_{02} \in \bar{\bar{\mathcal{N}}}_1 \cap \bar{\bar{\mathcal{N}}}_2$, then $x_{01}, x_{02} \in \bar{\bar{\mathcal{N}}}_1$ and $x_{01}, x_{02} \in \bar{\bar{\mathcal{N}}}_2$.

$$\begin{aligned}\alpha^I{}_{\bar{\bar{\mathcal{N}}}_1 \cap \bar{\bar{\mathcal{N}}}_2}(x_{01} \bowtie x_{02}) &= rmin\{\alpha^I{}_{\bar{\bar{\mathcal{N}}}_1}(x_{01} \bowtie x_{02}), \alpha^I{}_{\bar{\bar{\mathcal{N}}}_2}(x_{01} \bowtie x_{02})\}\\ &\gg rmin\left\{rmin\{\alpha^I{}_{\bar{\bar{\mathcal{N}}}_1}(x_{01}), \alpha^I{}_{\bar{\bar{\mathcal{N}}}_1}(x_{02})\}, rmin\{\alpha^I{}_{\bar{\bar{\mathcal{N}}}_2}(x_{01}), \alpha^I{}_{\bar{\bar{\mathcal{N}}}_2}(x_{02})\}\right\}\\ &= rmin\left\{rmin\{\alpha^I{}_{\bar{\bar{\mathcal{N}}}_1}(x_{01}), \alpha^I{}_{\bar{\bar{\mathcal{N}}}_2}(x_{01})\}, rmin\{\alpha^I{}_{\bar{\bar{\mathcal{N}}}_1}(x_{02}), \alpha^I{}_{\bar{\bar{\mathcal{N}}}_2}(x_{02})\}\right\}\end{aligned}$$

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$$= rmin\{\alpha^I_{\bar{\bar{N}}_1\cap\bar{\bar{N}}_2}(x_{01}), \alpha^I_{\bar{\bar{N}}_1\cap\bar{\bar{N}}_2}(x_{02})\}$$

$$\begin{aligned}\beta^I_{\bar{\bar{N}}_1\cap\bar{\bar{N}}_2}(x_{01} \bowtie x_{02}) &= rmin\left\{\beta^I_{\bar{\bar{N}}_1}(x_{01} \bowtie x_{02}), \beta^I_{\bar{\bar{N}}_2}(x_{01} \bowtie x_{02})\right\} \\ &\gg rmin\left\{rmin\left\{\beta^I_{\bar{\bar{N}}_1}(x_{01}), \beta^I_{\bar{\bar{N}}_1}(x_{02})\right\}, rmin\left\{\beta^I_{\bar{\bar{N}}_2}(x_{01}), \beta^I_{\bar{\bar{N}}_2}(x_{02})\right\}\right\} \\ &= rmin\left\{rmin\left\{\beta^I_{\bar{\bar{N}}_1}(x_{01}), \beta^I_{\bar{\bar{N}}_2}(x_{01})\right\}, rmin\left\{\beta^I_{\bar{\bar{N}}_1}(x_{02}), \beta^I_{\bar{\bar{N}}_2}(x_{02})\right\}\right\} \\ &= rmin\left\{\beta^I_{\bar{\bar{N}}_1\cap\bar{\bar{N}}_2}(x_{01}), \beta^I_{\bar{\bar{N}}_1\cap\bar{\bar{N}}_2}(x_{02})\right\},\end{aligned}$$

$$\begin{aligned}\gamma^I_{\bar{\bar{N}}_1\cap\bar{\bar{N}}_2}(x_{01} \bowtie x_{02}) &= rmax\left\{\gamma^I_{\bar{\bar{N}}_1}(x_{01} \bowtie x_{02}), \gamma^I_{\bar{\bar{N}}_2}(x_{01} \bowtie x_{02})\right\} \\ &\gg rmax\left\{rmax\left\{\gamma^I_{\bar{\bar{N}}_1}(x_{01}), \gamma^I_{\bar{\bar{N}}_1}(x_{02})\right\}, rmax\left\{\gamma^I_{\bar{\bar{N}}_2}(x_{01}), \gamma^I_{\bar{\bar{N}}_2}(x_{02})\right\}\right\} \\ &= rmax\left\{rmax\left\{\gamma^I_{\bar{\bar{N}}_1}(x_{01}), \gamma^I_{\bar{\bar{N}}_2}(x_{01})\right\}, rmax\left\{\gamma^I_{\bar{\bar{N}}_1}(x_{02}), \gamma^I_{\bar{\bar{N}}_2}(x_{02})\right\}\right\} \\ &= rmax\left\{\gamma^I_{\bar{\bar{N}}_1\cap\bar{\bar{N}}_2}(x_{01}), \gamma^I_{\bar{\bar{N}}_1\cap\bar{\bar{N}}_2}(x_{02})\right\}.\end{aligned}$$

Hence $\bar{\bar{N}}_1 \cap \bar{\bar{N}}_2$ is an interval valued picture fuzzy BG-Subalgebra of $\mathcal{X}$. Theorem 3.7. can be generalizes as follows.

Theorem 3.8. Let $\{\bar{\bar{N}}_i: i = 1,2,3 \ldots \ldots\}$ be a family of interval valued picture fuzzy BG-subalgebra of $\mathcal{X}$. Then $\bigcap \bar{\bar{N}}_i$ is also an interval valued picture fuzzy BG-Subalgebra of $\mathcal{X}$.

Theorem 3.9. An interval-valued picture fuzzy set $\bar{\bar{N}} = (\alpha^I, \beta^I, \gamma^I)$ is an interval valued picture fuzzy BG-Subalgebra of $\mathcal{X}$ if and only if the fuzzy sets α^I, β^I and ${\gamma^I}^c$ are interval valued fuzzy subalgebra of $\mathcal{X}$.

Proof: Let $\bar{\bar{N}} = (\alpha^I, \beta^I, \gamma^I)$ be a interval valued picture fuzzy BG-Subalgebra of $\mathcal{X}$ then we have
$\alpha^I(x_{01} \bowtie x_{02}) \gg rmin\{\alpha^I(x_{01}), \alpha^I(x_{02})\}$,
$\beta^I(x_{01} \bowtie x_{02}) \gg rmin\{\beta^I(x_{01}), \beta^I(x_{02})\}$ and
$\gamma^I(x_{01} \bowtie x_{02}) \ll rmax\{\gamma^I(x_{01}), \gamma^I(x_{02})\}$ for all $x_{01}, x_{02} \in \mathcal{X}$.
Clearly α^I, β^I are fuzzy subalgebra of $\mathcal{X}$.

Now $1 - \gamma^I(x_{01} \bowtie x_{02}) \gg 1 - rmax\{\gamma^I(x_{01}), \gamma^I(x_{02})\}$

$$\Rightarrow {\gamma^I}^c(x_{01} \bowtie x_{02}) \gg rmin\{1 - \gamma^I(x_{01}), 1 - \gamma^I(x_{02})\}$$
$$= rmin\{{\gamma^I}^c(x_{01}), {\gamma^I}^c(x_{02})\}.$$

Hence ${\gamma^I}^c$ is an interval valued fuzzy subalgebra of $\mathcal{X}$.

Conversely, assume that α^I, β^I and ${\gamma^I}^c$ are interval valued fuzzy subalgebra of $\mathcal{X}$.
For every $x_{01}, x_{02} \in \mathcal{X}$ we have $\alpha^I(x_{01} \bowtie x_{02}) \gg rmin\{\alpha^I(x_{01}), \alpha^I(x_{02})\}$, $\beta^I(x_{01} \bowtie x_{02}) \gg rmin\{\beta^I(x_{01}), \beta^I(x_{02})\}$ and ${\gamma^I}^c(x_{01} \bowtie x_{02}) \gg rmin\{{\gamma^I}^c(x_{01}), {\gamma^I}^c(x_{02})\} \Rightarrow 1 - {\gamma^I}^c(x_{01} \bowtie x_{02}) \ll 1 - rmin\{{\gamma^I}^c(x_{01}), {\gamma^I}^c(x_{02})\} = rmax\{1 - {\gamma^I}^c(x_{01}), 1 - {\gamma^I}^c(x_{02})\} \Rightarrow \gamma^I(x_{01} \bowtie x_{02}) \ll rmax\{\gamma^I(x_{01}), \gamma^I(x_{02})\}$.
Hence $\bar{\bar{N}} = (\alpha^I, \beta^I, \gamma^I)$ is an interval valued picture fuzzy BG-Subalgebra of $\mathcal{X}$.

Definition 3.10. Let $\bar{\bar{N}} = (\alpha^I, \beta^I, \gamma^I)$ is an interval valued picture fuzzy set defined on $\mathcal{X}$.
The operators $\circledast \bar{\bar{N}}$ *and* $\circledcirc \bar{\bar{N}}$ are defined as $\circledast \bar{\bar{N}} = (\alpha^I, \beta^I, {\alpha^I}^c)$ and $\circledcirc \bar{\bar{N}} = ({\gamma^I}^c, \beta^I, \gamma^I)$ respectively.

Theorem 3.11. If $\bar{\bar{N}} = (\alpha^I, \beta^I, \gamma^I)$ be an interval valued picture fuzzy BG-Subalgebra of $\mathcal{X}$ then $(i) \circledast \bar{\bar{N}}$ $(ii) \circledcirc \bar{\bar{N}}$ both are interval valued picture fuzzy BG-Subalgebras.

Proof: (i) It is sufficient to show that ${\alpha^I}^c$ satisfies the condition (PFBG-SA 3).

Let $\mathcal{x}_{01}, \mathcal{x}_{02} \in \mathcal{X}$ then we have

$$\alpha^I(\mathcal{x}_{01} \bowtie \mathcal{x}_{02}) \gg rmin\{\alpha^I(\mathcal{x}_{01}), \alpha^I(\mathcal{x}_{02})\}$$
$$\Rightarrow 1 - \alpha^I(\mathcal{x}_{01} \bowtie \mathcal{x}_{02}) \ll 1 - rmin\{\alpha^I(\mathcal{x}_{01}), \alpha^I(\mathcal{x}_{02})\}$$
$$\Rightarrow \alpha^{I^c}(\mathcal{x}_{01} \bowtie \mathcal{x}_{02}) \ll rmax\{1 - \alpha^I(\mathcal{x}_{01}), 1 - \alpha^I(\mathcal{x}_{02})\}$$

$= rmax\{\alpha^{I^c}(\mathcal{x}_{01}), \alpha^{I^c}(\mathcal{x}_{02})\}$.

Hence $\circledast\, \bar{\bar{\mathcal{N}}} = (\alpha^I, \beta^I, \alpha^{I^c})$ is an Interval valued picture fuzzy BG-Subalgebra of $\mathcal{X}$.

(ii) It is sufficient to show that γ^{I^c} satisfies the condition (PFBG-SA 1).

Let $\mathcal{x}_{01}, \mathcal{x}_{02} \in \mathcal{X}$ then we have

$$\gamma^I(\mathcal{x}_{01} \bowtie \mathcal{x}_{02}) \ll rmax\{\gamma^I(\mathcal{x}_{01}), \gamma^I(\mathcal{x}_{02})\}$$
$$\Rightarrow 1 - \gamma^I(\mathcal{x}_{01} \bowtie \mathcal{x}_{02}) \gg 1 - rmax\{\gamma^I(\mathcal{x}_{01}), \gamma^I(\mathcal{x}_{02})\}$$
$$\Rightarrow \gamma^{I^c}(\mathcal{x}_{01} \bowtie \mathcal{x}_{02}) \gg rmin\{1 - \gamma^I(\mathcal{x}_{01}), 1 - \gamma^I(\mathcal{x}_{02})\}$$

$\Rightarrow \gamma^{I^c}(\mathcal{x}_{01} \bowtie \mathcal{x}_{02}) \gg rmin\{\gamma^{I^c}(\mathcal{x}_{01}), \gamma^{I^c}(\mathcal{x}_{02})\}$.

Hence $\circledcirc\, \bar{\bar{\mathcal{N}}} = (\gamma^{I^c}, \beta^I, \gamma^I)$ is an Interval valued picture fuzzy BG-Subalgebra of $\mathcal{X}$.

Theorem 3.12. Let $\bar{\bar{\mathcal{N}}} = (\alpha^I, \beta^I, \gamma^I)$ be an interval valued picture fuzzy BG-Subalgebra of $\mathcal{X}$ then the sets

$\bar{\bar{\mathcal{N}}}_1 = \{\mathcal{x}_{01} \in \mathcal{X}: \alpha^I(\mathcal{x}_{01}) = \alpha^I(0)\}$,

$\bar{\bar{\mathcal{N}}}_2 = \{\mathcal{x}_{01} \in \mathcal{X}: \beta^I(\mathcal{x}_{01}) = \beta^I(0)\}$, and

$\bar{\bar{\mathcal{N}}}_3 = \{\mathcal{x}_{01} \in \mathcal{X}: \gamma^I(\mathcal{x}_{01}) = \gamma^I(0)\}$ are subalgebras of $\mathcal{X}$.

Proof: Let $\mathcal{x}_{01}, \mathcal{x}_{02} \in \bar{\bar{\mathcal{N}}}_1$. Then $\alpha^I(\mathcal{x}_{01}) = \alpha^I(0) = \alpha^I(\mathcal{x}_{02})$ and so $\alpha^I(\mathcal{x}_{01} \bowtie \mathcal{x}_{02}) \gg rmin\{\alpha^I(\mathcal{x}_{01}), \alpha^I(\mathcal{x}_{02})\} = rmin\{\alpha^I(0), \alpha^I(0)\} = \alpha^I(0) \Rightarrow \alpha^I(\mathcal{x}_{01} \bowtie \mathcal{x}_{02}) \gg \alpha^I(0)$. By using proposition 3.3 $\alpha^I(\mathcal{x}_{01} \bowtie \mathcal{x}_{02}) = \alpha^I(0) \Rightarrow \mathcal{x}_{01} \bowtie \mathcal{x}_{02} \in \bar{\bar{\mathcal{N}}}_1$.

Let $\mathcal{x}_{01}, \mathcal{x}_{02} \in \bar{\bar{\mathcal{N}}}_2$. Then $\beta^I(\mathcal{x}_{01}) = \beta^I(0) = \beta^I(\mathcal{x}_{02})$ and so $\beta^I(\mathcal{x}_{01} \bowtie \mathcal{x}_{02}) \gg rmin\{\beta^I(\mathcal{x}_{01}), \beta^I(\mathcal{x}_{02})\} = rmin\{\beta^I(0), \beta^I(0)\} = \beta^I(0) \Rightarrow \beta^I(\mathcal{x}_{01} \bowtie \mathcal{x}_{02}) \gg \beta^I(0)$. By using proposition 3.3 $\beta^I(\mathcal{x}_{01} \bowtie \mathcal{x}_{02}) = \beta^I(0) \Rightarrow \mathcal{x}_{01} \bowtie \mathcal{x}_{02} \in \bar{\bar{\mathcal{N}}}_2$. Let $\mathcal{x}_{01}, \mathcal{x}_{02} \in \bar{\bar{\mathcal{N}}}_3$. Then $\gamma^I(\mathcal{x}_{01}) = \gamma^I(0) = \gamma^I(\mathcal{x}_{02})$ and so $\gamma^I(\mathcal{x}_{01} \bowtie \mathcal{x}_{02}) \ll rmax\{\gamma^I(\mathcal{x}_{01}), \gamma^I(\mathcal{x}_{02})\} = rmax\{\gamma^I(0), \gamma^I(0)\} = \gamma^I(0) \Rightarrow \gamma^I(\mathcal{x}_{01} \bowtie \mathcal{x}_{02}) \ll \gamma^I(0)$. By using proposition 3.3 $\gamma^I(\mathcal{x}_{01} \bowtie \mathcal{x}_{02}) = \gamma^I(0) \Rightarrow \mathcal{x}_{01} \bowtie \mathcal{x}_{02} \in \bar{\bar{\mathcal{N}}}_3$. Hence the sets $\bar{\bar{\mathcal{N}}}_1, \bar{\bar{\mathcal{N}}}_2$ and $\bar{\bar{\mathcal{N}}}_3$ are BG-Subalgebras of $\mathcal{X}$.

REFERENCES

[1].[C. B. Kim and H. S. Kim, On BG-algebras, Demonstratio Mathematica,41 (2008), 497-505.

[2]. L. A. Zadeh, Fuzzy sets, Inform. and Control, 8 (1965), 338-353.

[3].S.S. Ahn and H.D. Lee, Fuzzy subalgebras of BG-algebras, Commun. Korean Math. Soc. 19(2) (2004), 243-251.

[4].K. T. Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets Syst.*, 20(1), (1986) 87–96, doi: https://doi.org/10.1016/S0165-0114(86)80034-3.

[5].B. Cuong, Picture fuzzy sets-first results. Part 1. In Proceedings of the Third World Congress on Information and Communication WICT'2013, Hanoi, Vietnam, 2013,1–6.

[6]. B. C. Cuong, V. Kreinovich, Picture fuzzy sets, J. Comput. Sci. Technol., 30 (2014), 409–420.

[7].P. Naga Sriveni, and K. Lalitha Parameswari, "Picture Fuzzy BG-Subalgebra in BG-algebra," *International Journal of Advance Research and Innovative Ideas in Education*, vol. 10, no. 5, pp. 250-255, Sep-Oct 2024. https://ijariie.com/AdminUploadPdf/Picture_Fuzzy_BG_Subalgebra_in_BG_algebra_ijariie24932.pdf?srsltid=AfmBOoqi9Kf4yLAd9e5MV76ppq_0Rty-G4zxslhkhizKzlJyzaEG8iSa